AEROELASTIC PROBLEMS ON AIRCRAFT CONSTRUCTION

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Abstract—The kinds of aeroelastic phenomena are described. The effects of these phenomena on aircraft construction are discussed. Developing a new aircraft several theoretical and experimental researches must be performed in order to avoid noxious aeroelastic phenomena during the later use of the aircraft.

1. INTRODUCTION

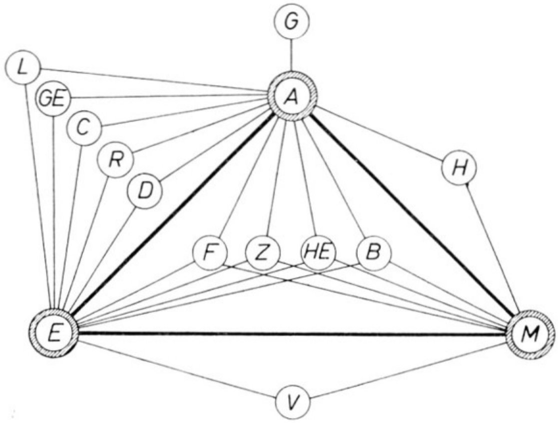
Noxious aeroelastic phenomena on aircraft are generally favored by increasing the flight speed and by diminishing the stiffness of the structural members. But both are trends of modern aircraft design. As the drag increases with the square of the frontal area at high speed, relatively thin wings and tail units are designed having a correspondingly low stiffness. Modern supersonic aircraft have, for instance, a wing thickness of only 4% of the wing chord. Such thin wings behave like elastic plates bending in all directions. For air speeds above Mach number 2 also, the aerodynamic heating must be considered.

The very high air speeds not only favor known aeroelastic phenomena, but raise new kinds of aeroelastic problems formerly not known. Therefore, to avoid noxious aeroelastic phenomena is today a more difficult task than only twenty years ago.

2. THE KINDS OF AEROELASTIC PHENOMENA

Aeroelastic phenomena arise if structural deformations induce additional aerodynamic forces. If they vary rapidly with time, also inertia forces are involved. Thence a large number of possible aeroelastic phenomena results, which I would like to explain by means of a triangle of forces (see Fig. 1).

The aerodynamic force A, the elastic force E, and the inertia force M are placed at the vertices of the triangle. Each phenomenon can be located on this diagram according to its relation to the three vertices. Below the triangle we have the ordinary mechanical vibrations without aerodynamic forces (V). Above the upper vertex we have the static stability of flight of a rigid aircraft without inertia (G). On the right-hand side we have the dynamic stability of flight of a rigid aircraft (H). These three regions do not properly belong to aeroelasticity, but are basic for it.



A	*Aerodynamic force*	*L*	*Load distribution*
E	*Elastic force*	*GE*	*Static stability of flight of elastic aircraft*
M	*Inertial force*	*C*	*Control effectiveness*
		R	*Control system reversal*
V	*Mechanical vibrations*	*D*	*Divergence*
G	*Static stability of flight*	*F*	*Flutter*
H	*Dynamic stability of flight*	*Z*	*Dynamic response*
		HE	*Dynamic stability of flight of elastic aircraft*
B	*Buffeting*		

FIG. 1. The aeroelastic triangle of forces.

The phenomena on the left-hand side of the triangle are so slow, that inertia forces can be neglected. Deformations of the flying aircraft induce additional aerodynamic forces on its surface; these induce again additional deformations. Generally a new stable equilibrium is attained. But the lift distribution is different from that one of a rigid aircraft, at first supposed on the design by the aerodynamicist. The bending moments of the wing may, for instance, be augmented by twist of the wing tips in flight (L). Such additional lift-distributions also induce pitching moments on the aircraft and therefore alter the static stability of flight (GE).

At the critical air speed certain aircraft have no stable equilibrium any more between the aerodynamic and elastic forces. The deformations augment aperiodically, until the wing or the tail unit are destroyed (D). A phenomenon of that kind is the static torsional instability of a slender wing (see Fig. 2). This phenomenon occurs if the center of twist line – – – is situated behind the aerodynamic center line —— —— of the wing. Therefore the aerodynamic forces produce a twisting moment augmenting the angles of attack at the airfoil sections and producing so even larger aerodynamic forces. For plate-like wings there exists aperiodic divergence by chordwise bending.

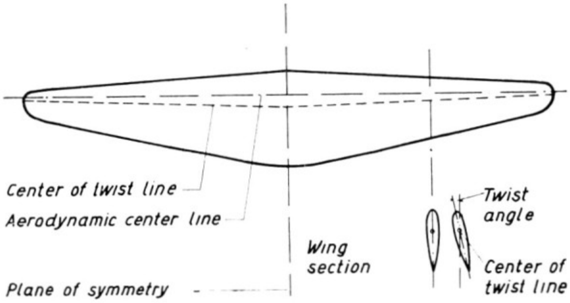


FIG. 2. Divergence of a slender wing.

Further on the wing twist can diminish the control effectiveness (*C*). The ailerons being moved by the pilot, an additional antisymmetric lift distribution is produced on both wings. This distribution tends to rotate the aircraft about its longitudinal axis, but at the same time it produces an antisymmetric wing twist inducing again additional aerodynamic forces. But these forces counteract to the rolling motion intended by the pilot. At the critical air speed the longitudinal moments induced by aileron turn and by subsequent wing twist cancel each other. Therefore the aircraft cannot be manoeuvred. At increased air speed the aileron effectiveness is even reversed (*R*).

Now we proceed to the phenomena inside the triangle. These are rapid motions also involving inertia forces. The most important phenomenon of this kind is flutter (*F*), being a self-excited oscillation of the aircraft, or parts of it. The necessary power to overcome the structural damping and the hinge friction at the oscillations is distracted from the air stream by the oscillatory motion itself. The amplitudes of oscillation may remain constant or may increase till destruction. This mainly depends on the increase or decrease of the damping loss angle with amplitude.

Flutter is a homogeneous dynamic stability problem without any external disturbances. The dynamic stability of flight of an elastic aircraft is a problem of the same kind, distinguishing from flutter mainly by the lower oscillatory frequencies (*HE*).

It is known that a rigid body has six degrees of freedom of motion in space. For an elastic body an infinite number of elastic degrees of freedom is added. At flutter the elastic degrees of freedom play the main part, while the six degrees of freedom of the rigid body motion only play a subordinate part. At dynamic stability of flight the parts are reversed. The static divergence (*D*) and the loss of control effectiveness (*R*) already mentioned are limiting cases of both phenomena at the oscillatory frequency zero.

In the free atmosphere there are regions with partial movements of the air having different causes and being denoted as gusts or atmospheric turbulence. If the aircraft is moving into such a region, there additional aerodynamic forces arise on it. Therefore the aircraft performs accelerated motions in space. These motions and deformations are denoted as the dynamic response of the aircraft to the known external forces (Z). This is mathematically a non-homogeneous dynamical problem. Phenomena of the same kind are the manoeuvres of the pilot and the running of the aircraft over a rough runway.

Turbulent disturbances may also arise on the aircraft itself, if its angle of attack increases till the boundary layer breaks away on certain spots of the surface. These disturbances firstly cause pressure variations on the spot of separation. By this, the so-called stall flutter on wings and airscrew blades may arise, the elastic oscillations of these members governing and increasing the detachment of the boundary layer.

Further the vortices of the separated flow may hit the tail unit and induce there violent irregular motions like atmospheric turbulence, denoted as buffeting (B).

The shocks arising on the wing and the fuselage, if locally the sound speed is attained, have similar effects. The shock may induce a breakaway of the boundary layer on this spot. At a profile thickness of at least 10% these shocks move periodically on the wing surface, producing large pressure variations and violent aileron oscillations. This phenomenon is denoted as "buzz oscillation" or, somewhat more dramatically, "hitting the sound barrier". But it can be avoided by a proper wing design.

The sound pressure of jet engines is so large, that the neighbored panels of the wing and the fuselage may be stressed up to the ultimate fatigue strength. In the sound field all frequencies are present. But the panel fastened on stiffening members is mainly excited in its lowest and least damped resonance frequency.

At supersonic air speed the panels of the wing and the fuselage may flutter. This phenomenon is favored by buckling of panels due to aerodynamic heating. Additional aerodynamic forces arise on the bump tending to press it into the opposite position until it jumps. Thereafter the event reverses. This is a non-linear flutter phenomenon causing destruction of the panel. Probably this phenomenon has, for the first time, occurred on the V–2 rocket. Recently it has also been observed on supersonic aircraft. At subsonic flow the pressure field on the bump has the opposite sign. It tends to enlarge the bump and could only cause aperiodic divergence at a correspondingly low panel thickness, but no panel flutter.

3. CONSTRUCTIONAL MEASURES TO PREVENT NOXIOUS AEROELASTIC PHENOMENA

The necessary constructional measures for preventing noxious aeroelastic phenomena have profound effects on the design of high-speed aircraft. Mainly the stiffness of the structural members is affected. But also the shape and the mass distribution of the wing, the tail unit, and the control surfaces are subjected to limiting conditions.

Stiffness Requirements

The aeroelastic instabilities have perhaps the most far-reaching effects of all aeroelastic phenomena. The aeroelastic instabilities of the wing mainly depend on its torsional stiffness. The torsional and bending stiffnesses of the fuselage have a large influence on the tail unit flutter, and have also a certain influence on the dynamic stability of flight.

All stiffnesses mainly depend on the material and on its temperature. Let us consider structural members of equal external shape and equal weight, but of different material. Let E be the modulus of elasticity, σ_B the ultimate tensile strength, and γ the specific weight of the material. Then the total stiffness is nearly proportional to E/γ, the buckling strength of panels with stiffening members is proportional to $\sqrt{(E)}/\gamma$, and the tensile strength is proportional to σ_B/γ. These coefficients are given in Table 1 for some known and future aircraft materials, cf. Refs. 3 and 4.

TABLE 1

Elastic coefficients of aircraft materials

Material	γ $\frac{\text{g}}{\text{cm}^3}$	E $10^6 \frac{\text{kg}}{\text{cm}^2}$	σ_B $10^3 \frac{\text{kg}}{\text{cm}^2}$	E/γ 10^9 cm	$\sqrt{(E)}/\gamma$ 10^3 c.g.s.	$\frac{\sigma_B}{\gamma}$ 10^6 cm
Beryllium	1·83	30·94	8·79	16·93	3·04	4·81
Magnesium	1·83	4·57	4·22	2·50	1·17	2·31
Aluminum	2·77	7·45	7·03	2·69	0·99	2·54
Titanium	4·71	11·74	12·66	2·50	0·73	2·69
Steel	7·83	20·39	21·09	2·60	0·58	2·69
Glass fibre	2·57	0·70	12·6	0·27	0·33	4·90
70% glass fibre 30% araldite	2·10	0·50	4·7	0·24	0·34	2·24
70% Be-wire 30% araldite	1·60	21·7	6·0	13·57	2·91	3·75

The metallic materials magnesium, aluminum, titanium, and steel used hitherto have nearly the same elastic coefficients at room temperature. Only with respect to buckling criteria is steel perceptibly inferior to the other materials.

Two new materials deserve a careful consideration. The metal beryllium has a six times higher stiffness coefficient than the four usual aircraft materials. Also with regard to buckling and tensile strength, it is three times and two times better. The greatest disadvantage is its brittleness, which may be improved in future by various promising methods.

The commercial glass fibre has a high ultimate tensile strength which can be improved up to 10^5 kg/cm^2 by special manufacturing methods.

But its modulus of elasticity is very low and can only be doubled by the addition of beryllium oxyde to the glass foundry. If one puts glass fibres into liquid epoxyde resin and then hardens it, a reinforced plastics is obtained with strength properties approaching those of metallic materials.

If only strength but not stiffness matters, this glass fibre plastics is suitable to manufacture complicated structural members in series at low costs. But if high stiffness of the structural members is required, the glass fibre plastics is not suitable, because it has only 10 to 20% of the stiffness of the usual materials.

If the glass fibres could be replaced by very thin beryllium wires, a reinforced plastics would be obtained being superior to the usual metallic materials at temperatures below 100°C. A wing composed of such a material would have nearly double the strength and the five-fold stiffness of an aluminum wing of equal weight. But it is doubtful if such a reinforced plastics may be produced economically.

If the aircraft shape and the material are given, the skin thickness of wing and fuselage is mainly determined by the required torsional stiffness. At the first design of an aircraft many possibilities have to be considered, and there is no time to perform aeroelastic calculations for all these possibilities. Therefore, especially in England, empirical formulas for the required torsional stiffness have been developed as follows:

$$\frac{\mathrm{d}M}{\mathrm{d}\theta} \geqslant \rho v_k{}^2 b t_m{}^2 K^2 \tag{1}$$

There M denotes a single twisting moment applied near the wing tip, θ twist angle there, b semi-span, t_m wing mean chord, ρ air density, and v_k critical air speed. The dimensionless coefficient K is an empirical function of several wing parameters, partly depending on aeroelastic calculations and partly on wind-tunnel tests of elastic-wing models.

The three critical air speeds of static torsional instability, loss of control effectiveness, and flutter depend on the design parameters in different ways. Figure 3 shows for instance its dependence on the sweep angle γ

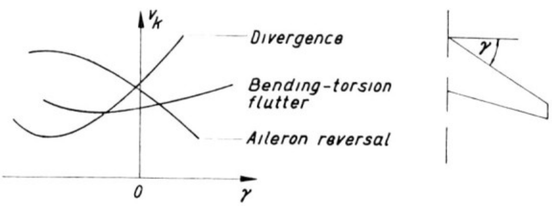


FIG. 3. Critical air speeds of a swept wing at various sweep angles γ.

of a swept wing. At small sweep angles the lowest critical air speed is given by flutter, at large sweep-back angles by loss of control effectiveness, and at large sweep-forward angles by static torsional instability.

Mass Distribution Requirements

Often considerable additional masses are fastened on wings; for instance, motors, fuel tanks, and arms. It matters at what places these masses are fastened. Figure 4 shows the results of British wind-tunnel tests on an elastic model of a swept-back wing, cf. Ref. 5. At first its critical flutter speed without additional masses has been measured and put equal with unity. Thereafter an additional mass has been fastened on different points of the wing, and the critical air speed has been measured again. In Fig. 4 the points of equal critical air speed are connected. The

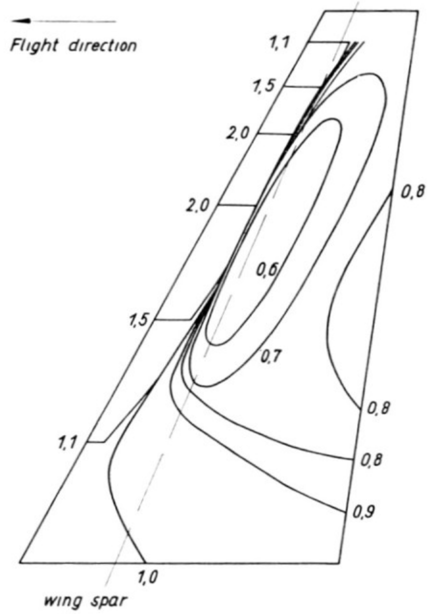


FIG. 4. Lines of equal critical air speed of flutter of a sweptback wing at different positions of an additional single mass $m = 1{\cdot}17$ wing mass.

highest critical air speed is obtained if the additional mass is fastened at two-thirds semi-span and before the aerodynamic center line.

On aeroelastic instabilities only a few elastic degrees of freedom mainly participate, having the lowest resonance frequencies. Out of the rigid degrees of freedom, often the rotations of control surfaces largely participate, especially on aircraft without servo-control, having practically free-turning control surfaces. The mass distribution of control surfaces is therefore mostly affected by aeroelastic requirements. The mass coupling of control surface rotation with the ground-bending mode of the wing or with the flapping and twisting modes of the tail unit is particularly noxious, because of their low resonance frequencies. For usual aircraft there exists an empirical upper limit of the reduced flutter frequency

$$\omega^* = \frac{\omega l_m}{v_k} < \omega_0^* \tag{2}$$

There ω is the circular oscillatory frequency, l_m is the half mean chord of wing or tail surfaces, and v_k is the critical air speed. The upper limits are approximately for wings $\omega_0^* = 1{\cdot}3$, for horizontal tail surfaces 0·55, and for vertical tail surfaces 0·40. Therefore, the lower the resonance frequency of the flutter mode, the lower the critical air speed.

By dynamical mass balance of the control surfaces the mass coupling between rotation of control surfaces and arbitrary bending and flapping oscillations of the wing and tail unit becomes zero. Such a measure particularly cancels the flutter possibilities having the critical air speed. By dynamical mass balance of a control surface I understand the coincidence of one of the three principal axes of its inertia ellipsoid with its hinge axis.

The control surfaces being easily rotatable about their hinge axes and being connected by their control cables and rods, are forming a separate oscillatory system. Such a system consists for instance of the two ailerons and of the control cables between them. If it happens that the resonance frequency of that system is very close to the resonance frequency of a symmetric wing oscillation, the reduced flutter frequency is raised up to its highest value ω_0^*, in spite of mass balance. The critical air speed is correspondingly decreased by such a "proximity of frequencies", which must be avoided.

From the aeroelastic point of view a bad aircraft design may be characterized by reduced flutter frequencies.

$$\omega^* = \omega_0^*$$

By a large weight penalty, resulting in high stiffnesses and resonance frequencies, it can, notwithstanding, be made safe, i.e. all critical air speeds being well above the largest flight speed.

In a good design, the designer intends to diminish the reduced frequencies ω^* of all possible flutter modes as much as possible, in order to save weight.

4. METHODS FOR SOLVING AEROELASTIC PROBLEMS AT THE DEVELOPMENT OF PROTOTYPE AIRCRAFT

Survey of the Different Methods

In order to avoid forbidding risks on a new type of aircraft, aeroelastic researches have to be performed from the first design up to the approval of the finished aircraft. There are mainly two methods for solving aeroelastic problems, namely the theoretical calculation and the experimental research of dynamically-similar models. Because the aircraft has a very complicated structure, in both cases simplifying assumptions are necessary in order to obtain results at all. Therefore, the aircraft manufacturers mostly apply both methods together, hoping that they correct each other. Finally, the air-worthiness of the finished aircraft must be proved by a flight resonance test.

The general aeroelastic problem can be solved in two steps. First, we can investigate the oscillations of the aircraft on the ground, determining its natural modes or elastic degrees of freedom. This can be done either theoretically or experimentally. It is appropriate to compare the results of different independent methods, in order to eliminate errors. In the second step we introduce these natural modes into the aeroelastic theory and calculate the corresponding aerodynamic forces. Out of it we finally obtain the critical air speed and the dynamic response of the aircraft.

This decomposition of the aeroelastic problem into two partial problems has already been tried in the past. But only during the last ten years has this method been accepted as the most useful, while electronic computers and electronic ground-resonance test technique have been applied to a large extent. Formerly the sufficiently accurate determination of natural modes failed either on computional difficulties or on deficiencies of the test technique.

Determination of the Ground Resonance Modes

The first step consists in the determination of the natural modes of the hovering aircraft without flow, especially for n elastic degrees of freedom $r = 1, 2, \ldots, n$. For each of these degrees of freedom the following four parameters are required:

$u_{ir}(P)$	natural mode or eigenfunction, i.e. oscillatory amplitude of an arbitrary point $P\ (x_1, x_2, x_3)$ of the aircraft in the direction of the co-ordinate x_i
ω_r	resonance frequency or eigenfrequency
$E_r = \frac{1}{2}\omega_r^2 M_r$	energy content of the oscillating aircraft
γ_r	loss angle due to structural damping (mean value).

Two arbitrary modes r and s are orthogonal; accordingly we have the important relation, cf. Ref. 6:

$$\sum_{i=1}^{3} \sum_{P} u_{ir}(P) u_{is}(P) m(P) = \begin{cases} 0, & \text{if } r \neq s \\ M_r, & \text{if } r = s \end{cases} \tag{3}$$

$m(P)$ denotes the mass at point P and M_r the generalized mass of the r-th degree of freedom. The second sum has to be taken over all mass points of the aircraft.

In addition to the n elastic degrees of freedom we have the six degrees of freedom of motion of the aircraft as rigid body and the degrees of freedom of rotation of the rigid control surfaces around their hinge axes. These have corresponding simple eigenfunctions $u_{ir}(P)$ and eigenfrequencies $\omega_r = 0$. Therefore we have to take into account N degrees of freedom altogether.

Using these degrees of freedom we may approximately represent an arbitrary harmonic oscillation of all points of the aircraft by

$$u_i(P, t) = \sum_{i=1}^{N} u_{ir}(P) q_r \exp(j\omega t) \tag{4}$$

There q_r denotes the generalized coordinate of the r-th degree of freedom, t the time and $j = \sqrt{-1}$.

A displacement of the points of the aircraft from their position of static equilibrium on the ground is only possible if external forces $K_i(P, t)$ act on several of these points. These external forces can be of any kind, for instance harmonical forces by electrodynamic shakers or aerodynamic forces. By means of the stiffness matrix, which I cannot explain here, one finally obtains the following important system of N linear equations for the generalized coordinates q_r, cf. Ref. 6:

$$q_r M_r[\omega_r^2 \exp(j\gamma_r) - \omega^2] + \sum_{i=1}^{3} \sum_{P} u_{ir}(P) K_i(P) = 0 \tag{5}$$

$$r = 1, 2, \ldots, N$$

These equations are called energy equations, because each term has the dimension of energy. According to this equation we have the resonance frequency of the damped system

$$\omega^2 = \omega_s^2 \cos \gamma_s \tag{6}$$

where ω_s denotes the resonance frequency of the undamped system. In most cases the loss angle γ_s is small; for metallic aircraft we have $\gamma_s = 0{\cdot}01$ to $0{\cdot}03$. Therefore $\cos \gamma_s \sim 1$. Then the resonance frequencies and the modes of the damped and undamped system are practically the same.

In order to calculate theoretically the natural modes and frequencies of the undamped aircraft, it is usually decomposed into a finite number of point masses. The elastic connecting members are represented by straight beams or flat plates having no mass. This representation is suitable for electronic computers. Sometimes it is called "beamology".

In order to excite corresponding pure modes on the ground resonance test, on each mass point $m(P)$ of the aircraft the excitatory force should be applied:

$$K_i(P) = m(P) u_{is}(P) \omega_s^2 j \sin \gamma_s \tag{7}$$

Indeed this is not possible. But as in the theoretical approach we can approximately decompose the aircraft into n masses $m(P_l)$ and apply an exciting force according to equation (7) in each center of gravity P_l. The results are impure modes $u_{is}(P)$ of first approximation.

It is important to exclude the $n-1$ neighboring modes from excitation putting the generalized coordinates

$$q_r = \begin{cases} 0, & \text{if } r = 1, 2, \ldots, n \neq s \\ 1, & \text{if } r = s \end{cases} \tag{8}$$

By equations (5), (6), (8) we obtain corrected excitatory forces $K_i(P_l)$, enabling a new and better experimental determination of the modes, and so on. The results are the better, the smaller the loss angle γ_s and the larger the number n of shakers.

At the ground resonance test the aircraft should dynamically behave like a hovering body. This can be done if the mass-spring-system of the elastic suspension is tuned to the resonance frequency ω_s, or if an additional electrodynamic shaker automatically compensates the alternating forces at the suspension point. Such a supporting device has also been used for the suspension of dynamically similar models in the wind tunnel.

Determination of the Critical Air Speed of Instability

The natural modes and frequencies of the aircraft on ground being determined, we can do the second step, namely introduce the aerodynamic forces on the surface of the aircraft as external forces $K_i(P)$ into equation (5). Generally these unsteady aerodynamic forces are given by integrals or by integral equations over the total surface of the aircraft, depending on the displacements $u_i(P')$ of all points of the surface and on their derivatives.

For slender wings we can approximately assume that the flow may be two-dimensional on all airfoil sections. Then we have only to integrate over one airfoil section at each time. This is the so-called strip theory, mostly used in aeroelasticity. At high supersonic air speed there exist simple approximate solutions assuming one-dimensional flow. This is the so-called piston theory being applicable to arbitrary shapes of wings and fuselage. Finally the aerodynamic forces can experimentally be determined by wind tunnel tests of oscillating wing models, cf. Ref. 7.

At very small displacements all these aerodynamic forces depend linearly on the generalized coordinates q_r. Therefore we finally obtain from equation (5) a system of N homogeneous linear equations for the generalized coordinates q_r. Such a system has finite solutions if and only if its denominatory determinant vanishes. This condition is called secular equation, which is an algebraic equation of Nth degree for the complex eigenvalues $\lambda = \omega^{-2}$, if the reduced frequency ω^* and the Mach number β are given. This secular equation has N complex roots

$$\lambda_s(\beta, \omega^*); \qquad s = 1, 2, \ldots, N \tag{9}$$

Flutter with a constant amplitude is only possible if at least one of these roots becomes real. Out of it the critical air speed, the critical frequency and mode of flutter or static divergence can be determined.

Both the natural modes and the flutter oscillations can experimentally be checked by tests on a dynamically similar model. Such a model must be geometrically similar; further, it must have the same mass distribution, the same stiffness matrix and the same specific weight as the prototype. If influences of the compressibility of the air shall be taken into account, also the ratios of the velocity of sound of the structural material and of the medium must be the same.

These requirements cannot fully be satisfied. Therefore, also, in model tests errors arise. Tests on the prototype aircraft are more reliable, particularly ground resonance tests and flight resonance tests. These test techniques have been considerably improved during the last time.

Aircraft Loads Due to Gusts

The ultimate strength of most structural members of aircraft is claimed either by gust loads or landing and runway roughness loads. Not the very high and very rare ultimate loads, but the more frequent median and small loads are mostly causing failures of structural members because of the fatigue of the material.

In order to determine the number and magnitude of the stress maxima of structural members, a statistical method is used, denoted as power spectral method. Along a straight flight path a random distribution of vertical gust velocities $w(s)$ may be given as function of the flight path s[km]. Then we have the autocovariance function, cf. Refs. 1 and 2:

$$R(s) = \lim_{s'=\infty} \frac{1}{2s'} \int_{-s'}^{s'} w(s')w(s + s')\, ds' \tag{10}$$

We define the following integral as the length constant of the turbulence:

$$L_0 = \frac{1}{R(0)} \int_0^{\infty} R(s)\, ds \tag{11}$$

$L_0 = 0{\cdot}15$ km is a common value. The Fourier-transforms of the function $R(s)$ is denoted as the power spectrum of turbulence:

$$\begin{aligned} \phi(\omega) &= \frac{2}{\pi} \int_0^{\infty} R(s) \cos \omega s\, ds \\ R(s) &= \int_0^{\infty} \phi(\omega) \cos \omega s\, d\omega \end{aligned} \tag{12}$$

The power spectrum may be known for a given gust field. Many measurements have been performed during the last years.

Let us firstly determine the stress originating in a given point P of the aircraft, if it is flying through a harmonical gust field of amplitude unity, given by the equation

$$w_1(s) = 1 . \exp j\omega s \tag{13}$$

This problem can be solved by calculating the disturbing aerodynamic forces $K'_i(P, \omega)$ on the aircraft due to this gust field, and inserting the sum

$$\sum_{i=1}^{3} \sum_{P} u_{ir}(P) K_i'(P, \omega) \tag{14}$$

into the right-hand side of equation (5). Out of this system of non-homogeneous linear equations the generalized coordinates $q_r(\omega)$ can be calculated. If a prototype aircraft is available for a ground-resonance test, the stress $\sigma_r(P)$ at the point P can be measured for each elastic degree of freedom r, using electric strain gages. We obtain the stress due to the harmonical gust field

$$T_1(\omega, P) = \sum_{r=1}^{N} q_r(\omega)\sigma_r(P) \tag{15}$$

This result is denoted as the dynamic response of the aircraft to the gust field equation (13). According to the statistical theory of linear systems we finally obtain the power spectrum of stresses due to the random gust field

$$\psi(\omega, P) = \phi(\omega)\left| T_1(\omega, P)\right|^2 \tag{16}$$

Assuming a Gaussian normal distribution, the number of stress maxima above a given stress σ_e can be calculated, occurring in the mean on a flight path of 1 km length:

$$\begin{aligned} N(\sigma_e, P) &= \frac{a_2}{2\pi a_0}\exp\left(-\frac{\sigma_e{}^2}{2a_0{}^2}\right)\ [km^{-1}] \\ a_0{}^2 &= \int_0^\infty \psi(\omega, P)\,\mathrm{d}\omega \\ a_2{}^2 &= \int_0^\infty \psi(\omega, P)\omega^2\,\mathrm{d}\omega \end{aligned} \tag{17}$$

These simple formulas are only valid if the wing span is small compared with the length constant L_0 of turbulence. Otherwise more complicated formulas occur, affording integration over the wing span. Further, two components of the gust velocity can be taken into account.

Before we can determine the fatigue life of the structural members of an aircraft, we have to know its flight route and flight level and the amount and strength of atmospheric turbulence on it during a year. Fatigue failures on structural members of aircraft due to gusts have been observed several times. They are not disastrous if the wing structure is statically undetermined to a high degree and if it is often inspected. But it seems to be better to avoid these failures at all.

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